1001. (a) Quote a theorem which tells you that $\left(x+\frac{2}{3}\right)$ must be a factor. Consider multiples of this.
 (b) Take out a factor of $(3x+2)$. You can use polynomial long division if needs be.

1002. List, and then restrict, the possibility space.

1003. Subtract y^2 from both sides, then divide.

1004. Call the factors on the LHS x and y, and rewrite the log statements as index statements.

1005. Test the endpoints to see whether they lie inside or outside the circle.

1006. Find the side lengths of $\triangle BXD$, which is a right-angled isosceles triangle.

1007. The vectors $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are perpendicular unit vectors. So visualise/draw the vectors as running along the edges of a unit cube.

1008. Show, using the discriminant, that the parabola $y = 10x - 21 - x^2$ doesn't intersect the line $y = x$. Then explain graphically why the parabola $y = 10x - 21 - x^2$ is always below $y = x$.

1009. (a) This is a standard congruency condition.
 (b) Use part (a).

1010. Write the graph as $y = ax^2+bx+c$. Then complete the square or differentiate.

1011. Consider reflection in the line $y = x$.

1012. Solve simultaneously by substituting for y, to find four such points.

1013. The graphs $y = x^{\frac{1}{2}}$ and $y = x^{\frac{1}{4}}$ are more closely related than $y = x^{\frac{1}{3}}$. Negative numbers have cube roots.

1014. Complete the square for each parabola. Find the vertices, then find the midpoint of the vertices.

1015. In each case, the simplified set is a single interval. Pay careful attention to the inclusion/exclusion of the endpoints. You may find it helpful to draw a number line.

1016. The census is the biggest possible sample. So, how big is it?

1017. Consider these as an AP. The formula for the sum of the first n terms is $S_n = \frac{n}{2}\big(2a + (n-1)d\big)$.

1018. A' is the complement of A, i.e. the negation, in set terms, of A: everything that is not in A.

1019. Express the fraction in lowest terms by factorising the numerator, then take the limit.

1020. The terms of the sum are the odd numbers, which are the shaded/unshaded chevrons.

1021. Expand and differentiate, setting the derivative to zero.

1022. In each case, use the word "negligible".

1023. Sketch and use circle geometry.

1024. Rearrange to $\mathrm{f}(x) = 0$ first. The factorise the LHS fully. You're looking for four angles.

1025. The height of the right-hand rectangle is n, and the width of the rectangles is 1.

1026. (a) Solve simultaneously for intersections, and then use calculus to compare gradients.
 (b) The distance (shortest distance) between two smooth curves has to lie along a path which is normal to both.

1027. Remember that any contact forces which are not frictional are reaction forces.

1028. Set up $\displaystyle\lim_{h\to 0} \frac{k\,\mathrm{f}(x+h) - k\,\mathrm{f}(x)}{h}$.

1029. Draw a sketch to locate the relevant points, find the equations of the diagonals, and solve simultaneously.

1030. Note that, in set notation, curly brackets signify a list or description of elements, not an interval.

1031. Draw a force diagram for the passenger, writing contact force as a single vector $\mathbf{C}$.

1032. Find the equation of the line, using the standard formula $y - y_1 = m(x - x_1)$. Then substitute $x = 0$. Alternatively, consider the change in y as x changes from $2a$ to a to 0.

1033. Call the lengths $x, 2x, 5x$ and set up an equation.

1034. On a Venn diagram, consider the extreme values: minimal overlap and maximal overlap between X and Y.

1035. The statement "$x = a$ is a root of $\mathrm{f}(x) = 0$" simply means that $\mathrm{f}(a) = 0$.

1036. This is a quadratic in a.

1037. Write b as $a^{\log_a b}$.

1038. Consider Newton's third law.

1039. With the standard formula $S = \pi(n-2)$, find the sum of the interior angles, then divide by 16.

1040. This is a geometric series. Use $S_\infty = \dfrac{a}{1-r}$.

1041. (a) Choose the first without loss of generality, and write down the probability that the second matches.
  (b) Consider the new restricted possibility space as consisting of the seven matched pairs.

1042. After using the binomial expansion, divide top and bottom by a before taking the limit.

1043. Solve $\tan\theta = 1$.

1044. The correlation coefficient r measures the extent to which bivariate data have a linear relationship.

1045. Find an expression for the angle between each line and the x axis.

1046. Let the trajectory start from O, with components of velocity v_x and v_y. Show that the equation of the trajectory is parabolic: you can then quote the fact that a parabola has a line of symmetry.

1047. The statement is false. Integrate both sides of $f'(x) = g''(x)$ to see why.

1048. The relevant derivative is $\frac{d}{dx}(\tan x) = \sec^2 x$.

1049. The possibility space consists of $6 \times 12 = 72$ equally likely outcomes. Alternatively, consider rolling the six-sided die first and looking for the probability that the twelve-sided die then matches it.

1050. Start by multiplying the top and bottom of the large fraction by x^2. Note that, in doing so, that you may introduce a new solution.

1051. Consider the x coordinates on a unit circle, or the y coordinates of points on the graph $y = \cos x$.

1052. Find the coordinates of the axis intercepts. Then split the kite into two triangles, one above and one below the x axis, and use the area formula $A = \frac{1}{2}bh$ for triangles.

1053. Express the first sentence as an equation, and then differentiate that equation with respect to x.

1054. $x \in P \implies x \in Q$ means: if x is an element of set P, then x is an element of set Q

1055. (a) Consider the y difference.
  (b) Consider the intersections of the curves.
  (c) Draw a sketch of $y = x^3 - x$ and $y = 3x$, Remember that an integral calculates *signed* area, not simply area.
  (d) Set up the calculation as a pair of integrals (or equivalently twice one integral), with $x = 0$ as one of the limits.

1056. Try e.g. $n = 1, 2, 3$.

1057. Take out a common factor of $(3x-2)(x-1)$.

1058. Apply the differential operator $\frac{d}{dx}$ to the three terms inside the bracket.

1059. The quadratic factor is always positive. So, you need the linear factor to be non-negative.

1060. The fourth line must be of the form $x + 2y = k$.

1061. Find $\frac{dy}{dx}$ for the parabola. Substitute this into the LHS of the differential equation, together with the original y. Simplify to reach the RHS.

1062. The boundary cases here are
  ① a polygon with side length 0 cm,
  ② a regular hexagon.

  Determine whether each of these is attainable.

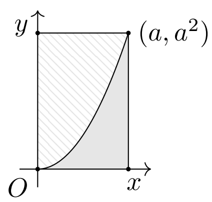
1063. Integrate the equation given.

1064. Use index laws to reshuffle the indices.

1065. Use log laws to write the index as a single log of the form $\ln(*)$. Then you can simplify as $e^{\ln(*)} = *$.

1066. By considering the hanging mass, write down the tension in the string. Then resolve parallel to the slope for the 4 kg mass.

1067. Without loss of generality, the scenario is



Use integration to find the shaded areas.

1068. A proof by exhaustion checks all possibilities. This doesn't necessarily mean writing them all down in a list, as different possibilities can be ruled out in different ways. Here, rule out all small pairs as too small, and check the large pairs.

1069. Use 3D Pythagoras. A unit vector has length 1.

1070. Set $b^2 - 4ac = 100$, and solve.

1071. Conditioning approach: consider the probabilities of the correct cards arriving as they are dealt; in this case the first card is a freebie (with probability of success 1).

——— Alternative Method ———

Combinatorial approach: the possibility space has ${}^{52}\mathrm{C}_5$ equally likely outcomes/hands.

1072. This is a disguised cubic. Take out a factor of $x^{\frac{1}{3}}$ first. Alternatively, use the substitution $z = x^{\frac{1}{3}}$.

1073. In both cases, sketch the graphs. In (a), this makes the answer obvious. In (b), you'll need to consider the quadratic discriminant.

1074. Apply the iteration twice to find B_3 in terms of a. Equate this to 18 and solve for a.

1075. The boundary equations are two concentric circles. Subtract the area of the inner from the area of the outer, noting that a and b are not squared.

1076. Show that $(x^2 - c^2 - 1)$ must have roots.

1077. Consider the fact that a resultant force of zero is also necessary for equilibrium.

1078. (a) Find the gradient of the given normal. Take the negative reciprocal to calculate a value m for the tangent at P. Differentiate $y = x^3$, and set up the equation $\frac{dy}{dx} = m$. Solve to find P.
(b) Sub the coordinates of P into the equation(s) of the normal.

1079. Any factorisation would have to be of the form

$$(x^2 + 1)(ax^3 + bx^2 + cx + d) \equiv 4x^5 + x + 1.$$

Multiply this out and equate coefficients.

1080. Draw a force diagram for the trapeze artist. One of the forces is the NIII pair of the force required. Solve NII to find it.

1081. Factorise the first equation as a quadratic in xy. Then rearrange to $y = ...$ and substitute into the second equation.

1082. Consider two stretches of the unit circle $x = \cos\theta$, $y = \sin\theta$, which has area π.

1083. Sketch a graph of $y = \mathrm{f}(x)$ on the domain $[a, b]$. Use the integral fact, remembering that definite integrals calculate *signed* area, not area.

1084. The diagonals of a rhombus are perpendicular.

1085. Both graphs are positive parabolae with vertices at (a, b).

(a) Rearrange to the form $y - b = m(x - a)^2$ and consider this as $y = x^2$, transformed.
(b) Rearrange to the form $x - b = m(y - a)^2$ and consider this as $x = y^2$, transformed.

1086. This can be seen algebraically by differentiating surface area $A = 2\pi rl$ with respect to time. Note that r is a constant.

1087. Since both points are outside the circle, this is equivalent to asking "Which point is closer to the origin?"

1088. Rewrite each logarithmic statement as an index statement.

1089. Sketch an example, e.g. a square and a hexagon.

1090. Solve the inequality $Z^2 > Z$ first. You'll get two intervals for Z. The probability of one of these can be written down by symmetry. For the other, use the normal distribution facility on a calculator.

1091. The gradient of the tangent is $2a$. So, the equation of the tangent is $y - y_1 = 2a(x - x_1)$.

1092. Show that the sequence is quadratic by finding the second differences. The general form is then $u_n = an^2 + bn + c$. So, use the fact that the second difference is $2a$ to find a. Then substitute in two values to find b and c.

1093. Consider the angle in a semicircle theorem.

1094. This is a pair of linear simultaneous equations in $\sqrt{x}$ and y^2. You can rename these X and Y to solve, or proceed directly.

1095. Draw a force diagram. It should have four forces on it, including a horizontal reaction force R. You should then consider the limiting case, in which the cube is in equilibrium, but on the point of slipping, i.e. friction is at $F_{\max} = \mu R$.

1096. The point $(1, a)$ must lie on the line. This allows you to calculate a. Then consider the gradient to find b.

1097. The fraction in the right-hand statement can only equal zero if it is well defined. Consider the root of the denominator. Your counterexample must also satisfy $p = q$.

1098. (a) The curve is a biquadratic, i.e. a quadratic in x^2. Factorise it as such.
  (b) Evaluate the derivative at the roots.
  (c) The curve is a positive quartic. The double (and not triple) roots are points of tangency (and not crossing) with the x axis.

1099. Translate the ratio information into an algebraic equation, and manipulate it.

1100. Set $\log_x y = a$. Rewrite this log statement as an index statement with y as the subject. Rearrange to make x the subject, and convert back into a log statement, writing $\log_y x$ in terms of a.

——— End of 11th Hundred ———